Introduction

Can you picture the dimensions of the solar system? Probably not. The sizes and distances involved are so great that the mind tends to give up, classifying the whole thing as “really, really big.” It’s even difficult to represent on paper! Let me explain what I mean:

- The above image shows the relative sizes of the Sun and its nine planets, but makes no attempt to represent the distances between them. Relative to their diameters, they’re really much farther apart than shown.

- The top diagram at right shows the true positions and orbital paths for the whole solar system. Can you tell how big each planet is from this view? What about the positions of the inner planets?

- This next image “zooms in” on the previous one by a factor of 18, showing the current state of the inner solar system. But now we’ve lost track of the outer planets!
You see? An 8.5” x 11” sheet of paper is just too small to represent visually all of the sizes and separations in our solar system. What about bigger paper? Well, blowing up the diagram of the outer solar system to match the scale of the inner solar system diagram shown above would require a poster 3 feet tall and 5 feet wide! And even then, the sizes of the planets still would not be represented.

So, how can we visualize our solar neighborhood? One way is to build a scale model. Take, for example, the model of a BMW Z4 Roadster shown below. This model is advertised as “1/43 scale,” which means that one inch on the model represents 43 inches (a bit over 3.5 feet) on the real car. This scale may also be written as a ratio, “1:43.” In order to design such a model, each and every feature of the real car must be measured and then scaled down by this same factor of 43. The model car will then be exactly 43 times smaller than the real thing, right down to the last detail.

Today you will be building two models at very different scales. One will be of the Earth-Moon system, from which you will be able to predict the angular size of Moon as viewed from here on Earth. The other will feature 8 of the 9 planets of our solar system, including their correct and current positions as they orbit around the Sun. From this model you will be able to predict what planets will be observable by us at night, and you will develop an observing schedule so that you won’t miss any observing opportunities during the evening lab tonight. It’s all up to you!

Before we get started, answer the following questions in your lab notebook. Don’t worry about being right or wrong, just honestly record your answers. You aren’t expected to know everything yet, we’re just getting started!

1. What real-world measurements do you think we will need to know in order to build our models?

2. How much smaller than the real thing do you think a “reasonable” model of the entire solar system would be? In other words, what scale should we use for our model? By “reasonable,” I mean that the planets should be large enough for us to see and manipulate, but the largest distances in the model shouldn’t be so large that it becomes cumbersome to build and interact with the model.

Part I: The Earth and the Moon

Most of us will spend our entire lives here on the planet Earth, catching occasional glimpses of our companion the Moon. (If you do get to go into space someday, send me a postcard!) Hundreds of astronauts and cosmonauts have orbited the Earth, but only 12 have ever set foot on the Moon, the last of whom were among the Apollo 17 crew back in 1972.

2003 Yerkes Summer Institute Day Lab 1 – Solar System Model
Our first “small step” out into the vastness of the solar system will be to discover just how we as human beings compare to the sizes and distances in the Earth-Moon system.

3. **Take a guess:** When Neil Armstrong became the first man to walk on the Moon back in 1969, how much farther from the surface of the Earth was he than one of today’s shuttle astronauts on a servicing mission to the Hubble Space Telescope?

Now, let’s begin to build a model of the Earth-Moon system. Your group will receive a ball of modeling clay, which you should divide into 50 equal pieces.

4. **Take a guess:** How many of these pieces of clay do you think should be used to construct your model of the Moon, if all of the remaining pieces must make up your model of the Earth? Record your answer. Build your model.

Your instructor will now tell you the actual ratio of the volumes of Earth and Moon. Remake your model according to this ratio.

5. How did your original model compare to this new model?

6. **Take a guess:** Based on intuition or previous knowledge, how far away do you think you should place the Moon from the Earth? Record your answer. Build your model. What was the reasoning behind your guess?

Now you will need the following facts: The diameter of the real Earth is 12,756 kilometers, and the true Earth-Moon separation is roughly 384,000 kilometers.

7. From the size of your model Earth and the diameter of the *real* Earth, use a calculator to determine the scale of your model. Then, calculate the appropriate separation between your Earth and Moon models. Record both answers. Correct your model.

8. How did your guess compare to the distance you’ve just calculated?

Now you will use your model to predict the angular size of the Moon as viewed from the surface of the Earth. In order to do so, we will use an equation with which you should be quite familiar by now, whether from our Tuesday prep sessions in July or from the other labs you’ve done here at the Summer Institute. I’m talking about the small-angle approximation in equal-fractions form:

\[
\frac{A}{360^\circ} = \frac{D}{2r}
\]

where \(A\) is the angular size of the Moon in degrees, \(D\) is the physical diameter of the Moon, and \(r\) is the distance between the observer and the Moon.

9. From the properties of your Earth-Moon model, calculate the angular size \(A\) of the model Moon as viewed from the surface of the model Earth. **Remember:** If you know the values of two variables from the small-angle approximation equation, you can calculate the value of the third variable.
Recall also that two triangles are called similar if the only difference between them is a constant scale factor between the lengths of corresponding sides, and that the corresponding angles of two similar triangles are by definition equal. Then by virtue of the constant scale factor between our model and the real Earth-Moon system, the angular size of the model Moon and that of the real Moon should be identical!

Having predicted the value of the angle $A$ from your model, you would like to confirm the prediction with a measurement of the true angular size of the Moon. However, we don’t have an accurate way to measure angles as small as what we expect to find here, so we will use a little trick. Instead of talking in terms of angles, we will instead use the properties of similar triangles. Namely, if you measure the apparent size of the Moon with a ruler mounted a specified distance from your eye, you will have constructed a third triangle that is similar to not only the one that encompasses the real Moon’s diameter, but also the related triangle within your model. You will make just such a measurement during the night lab tonight, but first we need to translate our angular expectations into similar-triangle-speak.

10. Based on the lunar-model diameter and Earth-Moon separation in your model, and using the properties of similar triangles, calculate the diameter of the Moon in millimeters, as viewed through a clear plastic ruler mounted at a specified distance from your eye.

Now we will insert the “human perspective” that I mentioned above. When a NASA space shuttle heads up on a reserving mission to the Hubble Space Telescope, it orbits at a height of about 600 kilometers above the surface of the Earth. This is the farthest above sea level that any human has been since the last manned lunar mission back in 1972. We will insert a toothpick into our model Earth, with the tip of the toothpick representing the height of the shuttle orbit.

11. Using the scale of your model, calculate the height of the space shuttle orbit within the model. Record your answer. Add a toothpick to your model to represent this height.

We are planning to build a full solar-system model anyway, so do you think it would be reasonable for us to continue using the current scale? To answer that question, consider what the Sun would look like in this model. The real Sun is an average of 149,598,000 kilometers from the Earth, and has a diameter of 1,392,000 kilometers.

12. Using the scale of your model, calculate the diameter of the model Sun you would need and the distance at which you would need to place it. Do you think we can keep using our current scale, or do we need to change it to move out farther into the solar system?

**Part II: The Solar System**

The answer to that last question, as you may have guessed, is that we need to drastically reduce the scale of our model in order to make it a reasonable size. Our constraints are that we need to be able to fit it on the Yerkes lawn, that we have unobstructed sight-lines from the Earth and Sun to all of the planets, and that the planet-models themselves not be so small as to be invisible.
The first step to creating a scale model is to measure all of the important properties of the full-sized object or system. If you were around for our Tuesday prep sessions in July, you know that we’ve already taken care of this step. We used NASA’s Solar System Simulator website (http://space.jpl.nasa.gov) to look up the distances of each planet from the Sun as of 9:30pm Central Daylight Time (CDT) on Monday, August 4th, 2003. This is the same as 2:30am the following day in Greenwich Mean Time (GMT, sometimes also called Universal Time or UT), which is a standard time astronomers and other scientists can agree upon no matter where they live. The image above is a sample output from the simulator, and as you can see, the “range” from the point of observation (the Sun) to Jupiter at that time is 803.009 million kilometers (803,009,000 km).

The list of these distances for all of the planets, as well as their diameters, is given in Table 1 below. Notice that most of the diameters are “only” between a thousand and one hundred thousand kilometers, while the distances range from 10 million to 5 billion kilometers. In general, these clumps of gas and/or rock are tiny compared to the distances that separate them!

<table>
<thead>
<tr>
<th>Celestial Body</th>
<th>Diameter (km)</th>
<th>Diameter (D_{Earth})</th>
<th>Distance from the Sun (km)</th>
<th>Distance from the Sun (A.U.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Sun</td>
<td>1,392,000</td>
<td>109.13</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Mercury</td>
<td>4,879</td>
<td>0.38</td>
<td>68,060,000</td>
<td>0.45</td>
</tr>
<tr>
<td>2. Venus</td>
<td>12,104</td>
<td>0.95</td>
<td>107,487,000</td>
<td>0.72</td>
</tr>
<tr>
<td>3. Earth</td>
<td>12,756</td>
<td>1.00</td>
<td>151,768,000</td>
<td>1.01</td>
</tr>
<tr>
<td>4. Mars</td>
<td>6,794</td>
<td>0.53</td>
<td>207,309,000</td>
<td>1.39</td>
</tr>
<tr>
<td>5. Jupiter</td>
<td>142,984</td>
<td>11.21</td>
<td>803,009,000</td>
<td>5.37</td>
</tr>
<tr>
<td>6. Saturn</td>
<td>120,536</td>
<td>9.45</td>
<td>1,351,000,000</td>
<td>9.03</td>
</tr>
<tr>
<td>7. Uranus</td>
<td>51,118</td>
<td>4.01</td>
<td>2,996,000,000</td>
<td>20.03</td>
</tr>
<tr>
<td>8. Neptune</td>
<td>49,528</td>
<td>3.88</td>
<td>4,500,000,000</td>
<td>30.08</td>
</tr>
<tr>
<td>9. Pluto</td>
<td>2,390</td>
<td>0.19</td>
<td>4,589,000,000</td>
<td>30.68</td>
</tr>
</tbody>
</table>

Table 1: Solar System Data

Because these numbers tend to be a bit overwhelming, we have also converted them into some more convenient units. The unit of choice for planet sizes is the Earth-diameter, which reveals that Jupiter’s diameter is about 10 times larger than the Earth’s, while the Sun’s is about 10 times larger still. For distances between planets we use the astronomical unit (A.U.), which is defined as the average distance between the Earth and the Sun. All of the inner, terrestrial planets are less than 1.5 A.U. from the Sun, while the farthest gas giant is a whopping 30 A.U. away!

Also, note that the Earth is currently about 1% farther from the Sun than the average. The orbits of all of the planets are elliptical, with some of them being more non-circular than others. You may find it surprising that we are actually farther away from the Sun during a Chicago summer than in winter!
It’s not our proximity to the Sun that makes it hotter and colder, but the slanting rays of its light caused by the tilt of the Earth’s axis. But that’s a story for another day.

One last set of data we need before we know what the real solar system looks like right now is the positions of the planets along their orbits around the Sun. We obtained these angles from the Solar System Simulator as well, using print-outs of the bird’s-eye-view diagrams for the outer and inner planets that are included on page 1 of this lab. We used a protractor to measure the angle centered at the Sun with the Sun-Uranus line representing 0°, and noted whether the planets were clockwise (+) or counterclockwise (-) from this line. Note that I have included this data in Table 2 below, but to make the construction of our model easier, I’ve converted it so that the Sun-Earth line now represents 0°, from which all angles are measured clockwise. Since the Earth was at 18.0° in the data you took, all I’ve done is to subtract that amount from all of your measurements.

Now that we know what the real solar system looks like on the night in question, we can consider what scale is appropriate for the construction of our model. Considering all of the constraints listed at the top of this section, the best scale seems to be about 110 times smaller than our Earth-Moon model, namely at a scale ratio of 1:16,360,000,000. The Earth from our previous model could represent the Sun in this one! One meter in our model will represent 16.36 million kilometers out in space, which is roughly 10 yards per astronomical unit. Using this scale, you calculated the scaled diameters and distances for the model, and these are listed along with the orbital positions in Table 2 below.

<table>
<thead>
<tr>
<th>Celestial Body</th>
<th>Diameter (mm)</th>
<th>Distance from the Sun (m)</th>
<th>Position in Orbit</th>
</tr>
</thead>
<tbody>
<tr>
<td>0. Sun</td>
<td>85.09</td>
<td>-------------------------</td>
<td>------------------</td>
</tr>
<tr>
<td>1. Mercury</td>
<td>0.30</td>
<td>4.16</td>
<td>+ 189.5°</td>
</tr>
<tr>
<td>2. Venus</td>
<td>0.74</td>
<td>6.57</td>
<td>+ 0.0°</td>
</tr>
<tr>
<td>3. Earth</td>
<td>0.78</td>
<td>9.28</td>
<td>+ 352.5°</td>
</tr>
<tr>
<td>4. Mars</td>
<td>0.42</td>
<td>12.67</td>
<td>+ 218.5°</td>
</tr>
<tr>
<td>5. Jupiter</td>
<td>8.74</td>
<td>49.08</td>
<td>+ 166.0°</td>
</tr>
<tr>
<td>6. Saturn</td>
<td>7.37</td>
<td>82.58</td>
<td>+ 342.0°</td>
</tr>
<tr>
<td>7. Uranus</td>
<td>3.12</td>
<td>183.13</td>
<td>+ 1.0°</td>
</tr>
<tr>
<td>8. Neptune</td>
<td>3.03</td>
<td>275.06</td>
<td>+ 53.5°</td>
</tr>
<tr>
<td>9. Pluto</td>
<td>0.15</td>
<td>280.50</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Properties of the Model

We are finally ready to begin construction. We will need a length of string for every Sun-planet separation listed in the table above. Lines of tape will be laid out on the Yerkes hallway floor to help you to measure the longest pieces of string (the distance to Neptune is equivalent to 3 football fields!). We will be using computer print-outs to represent the planets, the correct scale for which is shown in the image below. They will be affixed to placards on posts which we will plant at the appropriate places out on the lawn. We’ll use a 360° protractor, straight-pins, and corkboard to construct all of our angles.
13. As each new planet comes up for placement, ask yourself where you might expect it to be located based on what you’ve seen so far.

The first step is to place Uranus in its correct position, which is on the grass straight ahead of the Yerkes front steps. Facing the flagpole, we will then turn 30° east (clockwise) and use the Sun-Uranus string to find the correct location for the Sun. From here on out we will use the Sun as our base of operations, centering all of our angles there. Next up is to place the Earth 18° clockwise from the Sun-Uranus line. From now on, the Sun-Earth line will be our new zero-line for angle measurement. Next we will finish up the inner solar system with Mercury, Venus, and Mars, then work our way out to Jupiter, Saturn, and finally Neptune. Pluto will be left out of our model because it is inconveniently located deep within the neighboring golf course, but we will note the direction towards it from the Sun.

**Part III: Making Predictions**

Now that our model is complete, it’s time to see what it can teach us. By now you will have a true feel for the enormous distances separating these relatively tiny planets. But even some of these “tiny planets” are huge in their own right! (We don’t call them “gas giants” for nothin’.)

This model can do much more than just make us say, “Wow.” All of the information we’ve put into it so far has been centered at the Sun, but now we can walk the 9.28 meters to the Earth and see how the perspective changes. Some of the planets are very close to us at the moment, so they will be especially large and brilliant. Some others will be hiding behind the Sun or too close to it to be seen against the glare. You probably won’t be able to see many of the model planets’ disks with the naked eye, but with binoculars you can simulate what you’ll see during tonight’s night-lab.

14. The first step is to measure the angles – centered at the Earth – that separate each planet from the Sun. Record this data in your lab notebook.

15. From west to east (counterclockwise) and starting from the Sun, what is the order of the planets across the sky? This is the order in which they will rise and/or set.

Next we will attempt to understand the times at which each of the planets will rise and set tonight. Having already measured all of the planetary angles as observed from the Earth, we now have a handy pins-and-corkboard model of their locations in the night sky. If we align the 180° mark on the protractor with the pin that represents the Sun, and then mask off the bottom half of the protractor (180° - 360°), we will be able to represent the situation at sunset tonight. Every planet between 0° and 180° on the protractor will be visible in the sky at sunset, while all of those that are in the masked-off region will be “below the horizon.” Now we can rotate the protractor counterclockwise, simulating the rotation of the Earth. Notice that the Sun drops further below the horizon as we do this, just like it will in reality! Some planets will also set, while others will rise.

Now, the Earth rotates on its axis at a rate of 15° per hour, making a full rotation in 24 hours. (24 hr x 15°/hr = 360°.) We will assume that this constant rate applies to the apparent motion of the planets as well, and from this assumption we will predict rough rising and setting times for the planets. This is only an approximation, however, and due to the fact that the plane of the solar system is not aligned to
the Earth’s equator, these estimates could be wrong by as much as an hour. (To really get this right, we would need to rotate the protractor at different rates for different times of day.)

16. What planets, if any, do you think are too close to the Sun to be visible to you against the glare?

17. What planets, if any, do you think might be too far away from Earth to be visible?

18. What planets do you think will be visible to us at sunset or within 2 or 3 hours after?

19. What planets do you think would be visible if you were up later than that, but before sunrise? If there are any planets in this category, at what time would you need to be awake to observe them?

Now, take a look at where the Moon is in the sky right now, and remember where you’ve seen it over the past few nights. We will want to incorporate it into our plans for the evening…

20. As a group, your final task will be to plan your observing schedule for this evening’s night lab. Decide what priority each planet should have, based on when it will rise or set (you don’t want to be looking for something that isn’t up yet, or have something set before you’ve gotten to see it). If there are objects that will be up for the entire time, save them for when there are no other objects available. We will have access to binoculars as well as a telescope.

21. Propose your observing schedule for approval by your instructors.

Part IV: Testing Predictions at Night

22. For each object you try to observe, note the following information:
   - What equipment did you use?
   - If you couldn’t find that object, what do you think you would need to do for next time?
   - If you did find that object, was it where you expected?
   - What did you notice about the object?

23. Measure the angular size of the Moon. Does it match your predictions?

24. Observe the Moon in binoculars or a telescope. What do you notice? How big do you think the features are that you see on the surface?

Image Credits:
http://space.jpl.nasa.gov
http://www.pbase.com/image/16572540
http://www.ewal1.com/md/min21030.jpg
http://www.nasa.gov/ceps/rpif/img/earth/earthrise.gif
http://www.mapquest.com